Momentum confinement in DIII-D shots with impurities

W. M. Stacey
Georgia Institute of Technology, Fusion Research Center, Atlanta, Georgia 30332-0225

M. Murakami
Oak Ridge National Laboratory, Oak Ridge, Tennessee 37831

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A neoclassical momentum transport model, consisting of gyroviscous and convective components, is applied to the analysis of momentum confinement in DIII-D [Luxon, Anderson, Batty *et al.*, *Plasma Physics and Controlled Nuclear Fusion Research 1986* (IAEA, Vienna, 1987), Vol. 1, p. 159] experiments with significant impurity content. Good agreement between predicted and measured central rotation speeds and momentum confinement times is obtained, for L-mode (low-mode) discharges with and without neon injection and for an ELMing (edge-localized modes) H-mode (high-mode) discharge. The observed improvement in momentum confinement time with increasing neon impurity content in the L-mode shots can be accounted for by a neoclassical inward convective momentum flux that increases with impurity content. [DOI: 10.1063/1.1401115]

I. INTRODUCTION

As part of the investigation of impurity injection experiments in DIII-D,[1–2] the effect of impurity content on plasma rotation has been measured. Toroidal rotation speed and the associated momentum confinement time which can be inferred therefrom have been found to increase with increasing impurity content (as have the energy and particle confinement times). The causes for this increase in momentum confinement with increasing impurity concentration are of interest in themselves and may also provide some insights as to the causes for the observed increase in energy and particle confinement with increasing impurity content.

Momentum transport in tokamaks is widely held to be anomalous because both classical Braginski and neoclassical *perpendicular* transport rates are too small to account for momentum confinement times measured in tokamaks. However, the classical Braginski *gyroviscous* transport rate, when extended to toroidal geometry,[3] has been found[4] to be of the proper magnitude to account for measured momentum confinement times in a number of tokamaks.

Thus, we are motivated to test the predictions of a neoclassical momentum transport model, consisting of gyroviscous and convective components, against measured rotation velocities and inferred momentum confinement times for experiments in DIII-D with high impurity content. Our purposes in this paper are to briefly summarize the neoclassical momentum transport model and to present the results of this initial testing against DIII-D rotation measurements.

II. CALCULATION MODEL

A. Gyroviscous momentum transport

For our purposes, gyroviscous momentum transport across a flux surface in a tokamak for ion species "j" may be characterized by the frequency[3]

$$\nu_{dj}=\frac{T_j\theta_j r(L_{nj}^{-1}+L_{Tj}^{-1}+L_{v_{\phi}j}^{-1})}{2R_0^2 Z_j eB}, \tag{1}$$

a result which is the consequence of momentum diffusion across a flux surface, where

$$\theta_j=(4+n_j^c)\left[-\left(\frac{B_\phi}{B_\theta}\frac{v_{\theta j}}{v_{\phi j}}\right)(\Phi^s+n_j^s)+\Phi^s\right] + n_j^s\left[\left(\frac{B_\phi}{B_\theta}\frac{v_{\theta j}}{v_{\phi j}}\right)(2+\Phi^c+n_j^c)-\Phi^c\right] \tag{2}$$

($s=\sin, c=\cos$) must be evaluated on the flux surface, and the L_x are gradient scale lengths. The poloidal rotation speeds and sine and cosine components of the density and potential variation over the flux surface may be calculated[4] by taking the 1, $\sin\theta$, and $\cos\theta$ moments of the poloidal component of the momentum balance, for all ion species present and for the electrons.

B. Gyroviscous momentum confinement time

Using this expression for the toroidal momentum radial transport rate to evaluate the definition of momentum confinement time yields

$$\tau_\phi^{gv}\equiv\frac{2\pi R\Sigma_j\int_0^a\langle Rn_jm_jv_{\phi j}\rangle rdr}{2\pi R\Sigma_j\int_0^a\langle R^2\nabla\phi\cdot\nabla\cdot\Pi_j^{gv}\rangle rdr} = \frac{2R_0^2eB}{T_0}\;\frac{h_{nvT}}{h_{nv}}\;\frac{\left(\sum_j\frac{n_j}{n_e}m_j\right)_{av}}{\left(\sum_j\frac{n_j}{n_e}\frac{m_j}{Z_j}\theta_j r(L_{nj}^{-1}+L_{Tj}^{-1}+L_{v_\phi j}^{-1})\right)_{av}}, \tag{3}$$

where the h_{xy} are profile factors resulting from writing $n(r)=n_0 f_n(r)$, etc. For example,

$$h_{nv}^{-1} \equiv \frac{(2\pi R)\int_o^a f_n(r) f_v(r) 2\pi r dr}{(2\pi R)\int_o^a 2\pi r dr} = \frac{2}{a^2}\int_o^a f_n(r) f_v(r) r dr, \tag{4}$$

and the subscript av indicates an appropriate average of the quantity that may then be removed from under the integral to facilitate the approximate evaluation that is employed in this paper.

C. Convective and total momentum confinement times

There is also a convective momentum confinement time associated with the convective flux of angular momentum carried by the outward (or inward) particle flux. The total momentum confinement time may be written as

$$\tau_\phi^{\mathrm{th}} \equiv \frac{2\pi R \sum_j \int_0^a \langle R n_j m_j v_{\phi j}\rangle r dr}{2\pi R \sum_j \int_0^a \langle R^2 \nabla\phi \cdot \nabla \cdot \pi_j^{gv}\rangle r dr + 2\pi R \sum_j \int_0^a \langle R \nabla (n_j m_j v_{\phi j} v_{rj})\rangle r dr} = \frac{\tau_\phi^{gv}}{1+C}, \tag{5}$$

where the ratio of the gyroviscous to convective momentum confinement times is

$$C \equiv \frac{\int_0^a \sum_j \langle R \nabla (n_j m_j v_{\phi j} v_{rj})\rangle r dr}{\int_0^a \sum_j \langle R^2 \nabla \phi \cdot \nabla \cdot \Pi_j\rangle r dr} = \frac{\sum_j m_j (\Gamma_{rj})_{av}}{a \sum_j (n_j m_j \nu_{dj})_{av}}. \tag{6}$$

We calculate the convective particle fluxes needed to evaluate C from an extended neoclassical theory,[5] which includes beam momentum input, cross-field momentum transport, inertial and radial electric field effects, as well as the "standard" neoclassical collisional, parallel viscous, pressure gradient, and thermal friction effects.[6] This extended theory is summarized in Ref. 7.

D. Input neutral beam torque

The input torque from the neutral beams is related to the momentum confinement time and the angular momentum of the plasma by

$$\Gamma_\phi = \frac{(2\pi R)\int_0^a \left\langle R \sum_j^{\mathrm{ions}} n_j m_j v_{\phi j}\right\rangle 2\pi r dr}{\tau_\phi^{\mathrm{th}}}. \tag{7}$$

We use the TRANSP[8] calculation of torque for the calculations of this paper.

E. Central rotation frequency

The theoretical momentum confinement time of Eq. (3) and the neutral beam torque input can be combined to obtain an expression for the central rotation frequency,

$$\Omega_{\phi o}^{\mathrm{th}} \equiv \frac{v_{\phi o}^{\mathrm{th}}}{R} = \frac{\Gamma_\phi \tau_\phi^{\mathrm{th}} h_{nv}}{2\pi^2 a^2 R^3 \left(\sum_j^{\mathrm{ions}} \frac{n_j}{n_e} m_j\right)_{av} n_{eo}}. \tag{8}$$

F. Experimental momentum confinement time

Conversely, if the central toroidal rotation frequency and density and the rotation and density profiles are measured, then an experimental angular momentum confinement time can be constructed from an expression which is based on the same relationship among input torque, angular momentum, and momentum confinement time as in Eqs. (7) and (8), namely

$$\tau_\phi^{\exp} \equiv \frac{(2\pi R)\int_0^a \left\langle R \sum_j^{\mathrm{ions}} n_j m_j v_{\phi j}\right\rangle 2\pi r dr}{\Gamma_\phi} = \frac{2\pi^2 a^2 R^3 \left(\sum_j^{\mathrm{ions}} \frac{n_j}{n_e} m_j\right)_{av} n_{eo} \Omega_{\phi o}^{\exp}}{\Gamma_\phi h_{nv}}. \tag{9}$$

This expression, when evaluated with measured quantities, is the appropriate quantity for a consistent comparison with the theoretical expressions of Eqs. (3) and (8).

G. Evaluation of expressions

For the purposes of this comparison, measured experimental profiles of density, temperature, and toroidal rotation are used to evaluate the profile factors h_x. Measured central density n_{eo} and ion temperature T_{io} and measured impurity concentrations are used in evaluating the above expressions. The value of the torque and of the profile factors h_{nv} and h_{nvT} computed by TRANSP[8] are used.

TABLE I. A comparison of rotation speed and momentum confinement time for shots with varying impurity content and different confinement modes on DIII-D ($R=1.7$ m, $a=0.60$ m, $\kappa=1.7-1.9$, $E_b=80$ keV, $B=1.6$ T, $I=1.2$ MA).

	#99411 (1800 ms)	#98777 (1600 ms)	#98774 (1600 ms)	#98775 (1600 ms)
P_b (MW)	9.2	4.5	4.5	4.5
$n_{e0}(10^{20}\ \mathrm{m}^{-3})$	0.65	0.43	0.44	0.55
T_{i0} (keV)	8.0	3.5	4.9	6.4
n_{carbon}/n_e	0.050	0.011	0.007	0.005
n_{neon}/n_e	...	...	0.018	0.028
Conf. mode	H, ELMs	L	L	L
H_{89}	3.0	1.0	1.4	1.8
$V_{\phi 0}^{\exp}$ (10^5 m/s)	2.64	1.27	2.00	3.06
$V_{\phi 0}^{\mathrm{th}}$ (10^5 m/s)	2.90	1.52	2.22	2.90
$\tau_{\phi}^{\exp}(s)$	0.084	0.062	0.085	0.152
$\tau_{\phi}^{\mathrm{th}}(s)$	0.093	0.080	0.091	0.147
$\tau_{\phi}^{\mathrm{TRANSP}}(s)$	0.100	0.080	0.101	0.158
C [Eq. (6)]	−0.18	−0.05	−0.32	−0.60

Certain further simplifications are made. Flux surface geometry is approximated by toroidal geometry, and radial profiles are represented by a parabola to a power on a pedestal,

$$x(r)=(x_o-x_{\mathrm{ped}})\left(1-\left(\frac{r}{a}\right)^2\right)^{\alpha_x}+x_{\mathrm{ped}}, \tag{10}$$

with the α_x fit numerically to the measured profiles. In the absence of a pedestal, the profile factors h_{nv} and h_{nvT} take the simple form used in this paper,

$$\begin{aligned} h_{nv}&=1+\alpha_n+\alpha_v, \\ h_{nvT}&=1+\alpha_n+\alpha_v+\alpha_T. \end{aligned} \tag{11}$$

H. Solution procedure

The solution procedure for the gyroviscous momentum confinement time of Eq. (3) consists of (1) the calculation[4] of the poloidal rotation speeds and sine and cosine components of the densities and potential at $\rho=1/2$ from moments of the poloidal component of the momentum balance equations, which requires the toroidal rotation speeds as input; and (2) the calculation of the toroidal rotation speed, which requires the poloidal rotation speeds and sine and cosine components of the densities and potential as input in order to evaluate Eq. (3) to be used in evaluating Eq. (8). These two calculations are iterated to consistency.

Once the toroidal and poloidal rotation speeds are available, the radial electric field and neoclassical fluxes can be calculated.[5,7] The neoclassical particle fluxes are used in Eqs. (5) and (6) to calculate the convective correction C and thus the total momentum confinement time, which is then used in Eq. (8) to calculate the central rotation frequency.

In this paper, all ''av'' quantities, Eqs. (1) and (2), and the convective particle fluxes used in computing C are evaluated for the plasma properties at $r/a=1/2$.

III. THEORY/EXPERIMENT COMPARISONS

The investigation of L-mode shots in which the confinement is enhanced as a result of an increase in impurity concentration (i.e., RI mode) is a major area of research on DIII-D.[1,2] We consider three shots (98774, 98775, and 98777) from this study with a range of neon impurity concentrations and confinement enhancement. As a fourth case to test the theory on a very different type of shot, we choose an ELMing (edge-localized mode) H-mode shot (99411) with high carbon impurity content.

The results are shown in Table I. $V_{\phi 0}^{\exp}$ is the measured carbon rotation velocity at the center of the plasma, and $V_{\phi 0}^{\mathrm{th}}$ is the quantity calculated from Eq. (8). $\tau_{\phi}^{\exp}$ is the ''experimental'' quantity calculated from Eq. (9) using the measured $n_{e0}^{\exp}$, $V_{\phi 0}^{\exp}$ and experimental profiles to evaluate h_{nv}, and $\tau_{\phi}^{\mathrm{th}}$ is the ''theoretical'' quantity calculated from Eqs. (3), (5), and (6) using the measured profiles to evaluate h_{nv} and h_{nvT} and the measured T_{i0}. TRANSP[8] calculates the numerator (by interpretation) and denominator of the first form of Eq. (9) directly, without using the profile approximation of Eq. (10), thus providing a separate measure of the experimental momentum confinement time. This quantity, shown as $\tau_{\phi}^{\mathrm{TRANSP}}$ in the table, is in relatively good agreement with $\tau_{\phi}^{\exp}$, which is calculated the same way, but making use of the profile approximation (in both cases the total input torque from TRANSP is used).

The immediate conclusion is that the theoretical and experimental confinement times of Eqs. (5) and (9) agree quite well, given the approximations made in evaluating the momentum confinement time. The agreement of the measured central rotation speed and the theoretical value calculated from Eq. (8) follows from the agreement in momentum confinement time, since the experimental profiles are used in the evaluation of h_{nv} in Eq. (8).

A. Effect of neon injection on rotation and particle transport

Shots #98777, 98774, and 98775 are essentially identical discharges[1,2] which differ operationally only by the injection of different amounts of neon after 0.8 s in shots #98774 and 98775. It is noteworthy that the introduction of neon is predicted to produce an order of magnitude increase in the inward main ion flux and an inward neon flux of the same magnitude in shots #98774 and 98775, as calculated by neoclassical theory.[5–7] The larger quantity of neon injected in shot #98775 than in shot #98774 produces larger inward neoclassical particle fluxes. This effect results from the presence of the neon and its collisional interaction with the main ions, not from changes in profiles.

The observed increase in the rotation speed, relative to shot #98777 (w/o neon), when neon is injected in shots #98774 and 98775, is predicted rather well. This predicted increase in rotation speed is due almost entirely to the large increase in inward neoclassical convective momentum flux produced by the neon injection. The inward momentum convection produces a negative value of C which increases the total momentum confinement time of Eq. (5) relative to the gyroviscous momentum confinement time. The increases in momentum confinement time and in rotation speed are approximately linearly proportional to the concentration of the injected neon.

Although the extended neoclassical theory of Ref. 8 contains terms proportional to the beam momentum input, radial electric field, etc. that are not found in the conventional neoclassical theory, the enhancement of inward convection with neon injection was due to the conventional neoclassical terms.

An experimental value of the radial electric field was constructed from toroidal momentum balance using measured values of the toroidal and poloidal rotation speeds and of the pressure gradient, and a theoretical value was constructed in the same way using calculated toroidal and poloidal rotation speeds and pressure gradients calculated from the fitted experimental profiles. The experimental/theoretical values (kV) of the radial electric field were 26.0/20.7 for the no-neon shot #98777 and 38.3/33.7 for the 2.8% neon shot #98775. The increase in the value of the radial electric field with neon injection was measured to be 12.3 kV and predicted to be 13 kV.

B. High performance shot with large carbon concentration

Shot #99411 was a high performance ($H_{89}=3$) ELMing H-mode discharge with a 5% carbon concentration.[9] The agreement between the predicted and measured rotation speeds and momentum confinement times is as good as for the previous set of L-mode shots with quite different energy confinement characteristics.

C. Discussion

We tentatively conclude that the prediction of the neoclassical (gyroviscous plus convective) momentum confinement time given by Eqs. (3), (5), and (6), with the particle flux calculated according to Ref. 7, is in agreement with momentum confinement times measured in several impurity related experiments in DIII-D. This agreement spans the range $0.05 \leqslant |C| \leqslant 0.60$ of the ratio of the convective to gyroviscous momentum transport rates.

The large (up to an order of magnitude) increase in the inward main ion flux predicted for shots with neon injection or with high carbon concentrations (relative to a shot with no-neon and low carbon concentration) seems to be the principal factor that causes the predicted improvement in momentum confinement with increasing impurity concentration. Although the gyroviscous component dominates the momentum confinement time in all four shots, the difference in momentum confinement times in shots with and without neon is due primarily to the difference in the convective components of the momentum confinement time.

This increase in inward particle flux and the corresponding increase in inward convective energy flux with increasing impurity concentration are also qualitatively consistent with the experimental observation of improvement in particle and energy confinement with increasing impurity concentration.[1,2] The increase in inward particle flux with increasing impurity concentration is also in qualitative agreement with the results of transport simulations[10] that find that a larger inward main ion pinch term is necessary to model discharges with increased impurity concentrations. We intend to return to this issue and to a comparison of other neoclassical predictions of particle and energy transport in DIII-D impurity-related experiments in the near future.

APPENDIX: GYROVISCOUS MOMENTUM CONFINEMENT THEORY

Cross-field momentum transport in tokamaks is widely regarded as being "anomalous" because the familiar and physically intuitive perpendicular viscosity is too small by at least an order of magnitude to account for momentum confinement times observed experimentally. However, as shown by Braginski,[11] the development of a viscous stress tensor for charged particles in a magnetic field from the general strain tensor of fluid theory leads to two cross-field momentum transport processes, perpendicular viscosity and gyroviscosity.

Stacey and Sigmar[3] extended the Braginski formalism to toroidal flux surface geometry and demonstrated that the perpendicular viscosity arose from radial nonuniformities in the toroidal rotation frequency and that the gyroviscosity arose from poloidal nonuniformities over the flux surface in the toroidal rotation frequency. In particular, gyroviscosity was shown to depend on up–down nonuniformity over the flux surface. Subsequently, first-principle calculations of poloidal rotation and poloidal density nonuniformities over the flux surface were made which demonstrated that $<O(\epsilon)$ poloidal nonuniformities in both main ions and impurities lead to gyroviscous momentum transport rates that are sufficient to explain the experimentally observed momentum damping rates in a number of tokamaks.[4,12]

Mikhailovski and Typsin[13] also extended the Braginski viscosity formalism to toroidal geometry and to include drift velocities. They found a drift correction to the gyroviscosity formalism used in this paper[3] which substantially reduces the magnitude of the cross-field gyroviscous momentum flux when $v_{\varphi}/v_{\mathrm{th}} \leqslant \rho_{\theta}/L_p$, but this correction becomes neglible when $v_{\varphi}/v_{\mathrm{th}} \geqslant \rho_{\theta}/L_p$, as is the case in the core of strongly rotating plasmas such as are considered in this paper. Thus, as recently noted by Claassen *et al.*,[14] this drift correction will change the value of the cross-field gyroviscous momentum transport rate in the edge. However, it will have little if any effect on the transport rate from the center to the edge which determines the overall momentum confinement, hence little effect on the results of this paper.

We now comment briefly on two other contemporary developments of cross-field momentum transport theory which found that neoclassical effects do not enhance the perpendicular viscosity, but neither of which found a cross-field gyroviscous momentum transport term of the type found in Refs. 3 and 13. Both Hinton and Wong[15] and Connor *et al.*[16] found a cross-field momentum transport flux with a viscosity coefficient proportional to the collision frequency and equal in magnitude to the classical perpendicular viscosity coefficient of Braginski and driven by a radial gradient of the toroidal rotation velocity. Both sets of authors[15] used a gyroradius ordering which results, to leading order, in a poloidally uniform toroidal rotation frequency, hence $\theta_j = 0$, and $v_{\theta j} = 0$. At the next order, the gyroradius ordering (perturbation) theory yields nonzero $v_{\theta j}$ and θ_j of the same order of magnitude calculated in this paper.[17]

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